We note that except for the Lewis number, the expression for ϕ is the same as that inferred from Ref. 2 for inclusion of heat conduction from the droplet into the surrounding air, e.g.,

$$\phi = 1 + \rho_0 c_p T(kT/Mh_v)^2 p_v^{-1} Le^{-1} \quad (26)$$

If the Lewis number is less than unity, then the heat conduction is sufficient to prevent recondensation of the air in the heat conduction regime. However, the Lewis number for the diffusion of water vapor is about 1.5; hence the heat conduction from the droplet is not quite sufficient to prevent recondensation, contrary to Ref. 1. In the regime where heat conduction can be neglected, the heat absorbed by the droplet vaporization is definitely not sufficient to prevent recondensation.

If the regime of operation in Fig. 1 is such that conduction to the air is negligible, then $\phi = 1$. For the regime where conduction must be considered, ϕ is shown as a function of temperature for saturated air in Fig. 2.

The boundary conditions for Eq. (24) are

$$x = \infty, I = 0, \rho_f = \rho_{f\infty}, x = -\infty, I = I_0, \rho_p = 0 \quad (27)$$

and we arbitrarily set $I = \frac{1}{2}I_0$ at $x = 0$. The solution to Eqs. (24), subject to Eq. (27) is

$$\rho_f(x)/\rho_{f\infty} = 1 - I(x)/I_0 \quad (28)$$

$$I(x)/I_0 = [1 - \exp(\alpha\rho_f x/\rho_l)]^{-1} \quad (29)$$

$$u = I_0/\rho_f h_v \phi \quad (30)$$

Above the boiling line in Fig. 1, h_v must be increased by about 12% (at room temperature) to account for the additional energy to heat the droplet to its boiling point. In addition, it can be shown that for any given droplet

$$r(x)/r_\infty = [\rho_f(x)/\rho_{f\infty}]^{1/3} \quad (31)$$

Equations (29) and (31) are shown in Fig. 3.

The time-dependent solution of the equations in stationary coordinates x' been derived independently by Glickler.[7] His results are

$$I(x',t)I_0 = \{1 + [\exp(Z_0\alpha x') - 1]\exp(-\alpha t I_0/h_v\rho_l)\}^{-1} \quad (32)$$

Equation (32) reduces to the quasi-stationary solution, Eq. (29), in those parts of the fog where $I(x',0) \ll I_0$.

Scattering by Vaporized Droplet

After the droplet is vaporized, and before diffusion sets in, the vapor will occupy a spherical volume of radius $R \sim 10r$. This causes elastic scattering, whose cross section is[8]

$$Q_{sv} = 2\pi k^2(\Delta n)^2 R^4 \quad (33)$$

which decreases with time due to diffusion. For a typical fog droplet the time for diffusion to cause an appreciable change in Q_{sv} is

$$t \approx 10^{-2}[4N^{2/3}\mathfrak{D}]^{-1} \quad (34)$$

For example, if $N = 10^3$, $t \approx 10^{-4}$ sec.

Discussion

The profiles shown in Fig. 3 are somewhat different than those given in the past. For example if ρ_f is constant, Eq. (24) predicts that

$$I/I_0 = \exp(-\alpha\rho_f x/\rho_l) \quad (35)$$

We see that Eq. (29) has this behavior as $x \to \infty$. On the other hand, if the intensity is constant, then the fog density decreases exponentially with distance. Thus the solution, Eqs. (28) and (29), bridges the two limiting solutions of constant intensity and constant fog density.

It is instructive to consider the history of some typical droplets as they vaporize; to do this, we have plotted Eq. (31) on Fig. 1 for two typical initial drop sizes, 3 and 5 μm, for $I_0 = 10^6$ w/cm². The undisturbed fog is at the left; initially the droplet is in the heat conduction regime. As the beam propagates into the fog, the radius decreases and the droplet moves into the vaporization limit region, almost reaching the shock line. As the droplet radius decreases further, the droplet moves back into the heat conduction regime.

As an example, consider an extremely dense fog of 0.5×10^{-6} g/cm³. For a beam intensity of 10^6 w/cm², the propagation speed is about 6.7×10^6 M/sec. Consider a pulse time of 10^{-4} sec. The beam will propagate about 0.67 km. From Eq. (33) we may estimate the extinction length for the vaporized droplet, using $(n - 1) = 2.73 \times 10^{-4}$ for air and 1.83×10^{-4} for water vapor[9] at one atmosphere and 100°C, and assuming a Gaussian distribution of fog droplet initial radii. The result is that the extinction length is 7 km. Since this is much larger than the distance propagated, scattering by vaporized droplets is negligible. Since the vapor droplets diffuse after about 100 μsec, longer pulse times and fog dispersion lengths should be possible. An interesting question is the subsequent history of the vaporized fog droplets as they begin to recondense, but this question is beyond the scope of the present Note.

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Unsteady Radiation Slip

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Introduction

Although increased attention has been paid in the last decade to problems of radiating gas, the related boundary conditions so far appear to have received inadequate treatment. This is probably caused by the fact that the astrophysicist and the gas dynamicist are not primarily concerned with boundaries, and the boundary layer specialist has been often forced, because of the complexity of his problems, to consider only black surfaces. It is the purpose of this study to discuss conditions for diffuse nongray boundaries of un-

Received March 5, 1970; revision received June 18, 1970.
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steady radiating nongray gas, with particular emphasis on the slip condition. To keep the physics in the foreground and the mathematics simple, the discussion is retained one-dimensional.

As is well known, there are two approximate methods for the evaluation of one-dimensional radiant flux. One of these, the exponential approximation, integrates the transfer equation and evaluates the heat flux approximating the exponential integrals by exponentials. The other, the discrete streams, replaces the intensity distribution by a number of discrete intensities and employs the moments of transfer equation. Actually, the first-order flux obtained by these methods is known to satisfy the same differential equation with slightly different coefficients. In this study the first approximation of the latter, the two-stream distribution of intensities, is employed. The method was conceived by Schuster[1] and Schwarzchild[2] (extended to higher approximations by Wick[3] and Chandrasekhar[4] for the study of polarization and anisotropic scattering). The objectives of the present investigation are the unsteady monochromatic slip, the unsteady temperature jump, and an illustrative example that are, respectively, discussed below.

Monochromatic Slip

Consider the one-dimensional form of the transfer equation,

$$\mu\, \partial I_\nu/\partial\tau_\nu = \beta_\nu - I_\nu \tag{1}$$

where $\mu = \cos\phi$, ϕ being the angle between the intensity and the surface normal, and $\tau_\nu = \alpha_\nu x$, x denoting the coordinate normal to this surface. In terms of two-stream intensities, the first moment of Eq. (1) over the solid angle gives

$$\partial q_\nu{}^R/\partial\tau_\nu = 4E_{b_\nu} - 2\pi(I_\nu{}^+ + I_\nu{}^-) \tag{2}$$

where $E_{b_\nu} = \pi B_\nu$, E_{b_ν} being the monochromatic emissive power, and

$$q_\nu{}^R = \int_\omega \mu I_\nu d\omega = \pi(I_\nu{}^+ - I_\nu{}^-) \tag{3}$$

On the other hand, the second moment of Eq. (1), obtained multiplying this equation by $\mu d\omega$ and integrating the result over the solid angle, yields

$$(2\pi/3)\partial(I_\nu{}^+ + I_\nu{}^-)/\partial\tau_\nu = -q_\nu{}^R \tag{4}$$

From Eqs. (2) and (4) it now follows that

$$\partial^2 q_\nu{}^R/\partial\tau_\nu{}^2 - 3q_\nu{}^R = 4\partial E_{b_\nu}/\partial\tau_\nu \tag{5}$$

Next eliminating $I_\nu{}^-$ between Eqs. (2) and (3) gives for $\tau_\nu = 0$

$$\pi I_\nu{}^+(0+,t) = E_{b_\nu}(0+,t) + \tfrac{1}{2}q_\nu{}^R(0,t) - \tfrac{1}{4}\partial q_\nu{}^R(0,t)/\partial\tau_\nu \tag{6}$$

Also the definition of the monochromatic surface intensity (Fig. 1) leads, in view of Eq. (3), to

$$\pi I_\nu{}^+(0+,t) = E_{b_\nu}(-0,t) - (\rho_\nu/\epsilon_\nu)q_\nu{}^R(0,t) \tag{7}$$

Finally, inserting Eq. (7) into Eq. (6) the monochromatic radiation slip is found to be

$$E_{b_\nu}(-0,t) - E_{b_\nu}(0+,t) = (1/\epsilon_\nu - \tfrac{1}{2})q_\nu{}^R(0,t) - \tfrac{1}{4}\partial q_\nu{}^R(0,t)/\partial\tau_\nu \tag{8}$$

provided $\epsilon_\nu \neq 0$. For a mirror surface $\epsilon_\nu = 0$, and Eq. (8) reduces to $q_\nu{}^R(0,t) = 0$, which implies, equivalently,

$$E_{b_\nu}(-0,t) - E_{b_\nu}(0+,t) = -\tfrac{1}{4}\partial q_\nu{}^R(0,t)/\partial\tau_\nu \tag{9}$$

Appreciable conduction eliminates the slip, and Eq. (8) now becomes

$$0 = (1/\epsilon_\nu - \tfrac{1}{2})q_\nu{}^R(0,t) - \tfrac{1}{4}\partial q_\nu{}^R(0,t)/\partial\tau_\nu \tag{10}$$

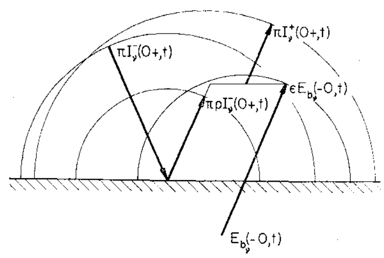

Fig. 1 Opaque surface intensity.

the result previously given by Goody.[5] Under steady conditions and in the absence of conduction, the balance of thermal energy simplifies to $\partial q_\nu{}^R/\partial\tau_\nu = 0$ and Eq. (8) assumes the form

$$E_{b_\nu}(-0) - E_{b_\nu}(0+) = (1/\epsilon_\nu - \tfrac{1}{2})q_\nu{}^R(0) \tag{11}$$

already obtained by Deissler,[6] Arpaci and Larsen,[7] and Finkleman,[8] however, for only the thick gas limit.

Temperature Jump

Here, in view of the recent studies, the nongray features may be incorporated into the discussion. Following Traugott,[9] the first moment given by Eq. (2) may be written in terms of the total values and the Planck mean of the absorption coefficient as

$$\partial q^R/\alpha_P\partial x = 4E_b - 2\pi(I^+ + I^-) \tag{12}$$

Clearly, $4E_b \gg 2\pi(I^+ + I^-)$ for thin gas, and Eq. (12) reduces to the well-known flux of thin gas, $\partial q^R/\partial x = 4\alpha_P E_b$, excluding the effect of boundaries. For large values of the absorption coefficient, Eq. (12) degenerates to $4E_b \to 2\pi(I^+ + I^-)$, the condition for thick gas. Next, the second moment given by Eq. (4) may be stated in terms of the total values and the Rosseland mean of the absorption coefficient as

$$(2\pi/3)\partial(I^+ + I^-)/\alpha_R\partial x = -q^R \tag{13}$$

which, in terms of $4E_b = 2\pi(I^+ + I^-)$, gives the well-known flux of thick gas, $q^R = -(4/3\alpha_R)\partial E_b/\partial x$. The combination of Eqs. (12) and (13) results in

$$(\partial/\alpha_R\partial x)\partial q^R/\alpha_P\partial x - 3q^R = (4/\alpha_R)\partial E_b/\partial x \tag{14}$$

This equation, first suggested by Traugott,[9] will later be modified by two recent references cited in the last section. Here, introducing $\zeta = (\alpha_P/\alpha_R)^{1/2}$, the geometric mean of the Planck and Rosseland coefficients, $\alpha_M = (\alpha_P\alpha_R)^{1/2}$, and $\tau = \alpha_M x$, Eq. (14) may be rearranged as

$$\partial^2 q^R/\partial\tau^2 - 3q^R = 4\zeta\partial E_b/\partial\tau \tag{15}$$

To proceed further, the balance of thermal energy is needed. Consider, for example, the simple case, the unsteady one-dimensional stagnant gas, governed by

$$\rho c_v\partial T/\partial t^+ = \alpha_M{}^2k\partial^2T/\partial\tau^2 - \alpha_M\partial q^R/\partial\tau \tag{16}$$

where t^+ denotes the time. Eliminating q^R between Eqs. (15) and (16) gives

$$(\partial^2/\partial\tau^2 - 3)\partial\theta/\partial t = \mathcal{P}_0(\partial^2/\partial\tau^2 - 3)\partial^2\theta/\partial\tau^2 - \zeta\,\partial^2\theta^4/\partial\tau^2 \tag{17}$$

where $\theta = T/T_0$, T_0 being a reference temperature, $t = 4\alpha_M\sigma T_0{}^3t^+/\rho c_v$ and $\mathcal{P}_0 = \alpha_M k/4\sigma T_0{}^3$. Clearly, as $\mathcal{P}_0 \to 0$, Eq. (17) reduces to

$$(\partial^2/\partial\tau^2 - 3)\partial\theta/\partial t = -\zeta\partial^2\theta^4/\partial\tau^2 \tag{18}$$

the case of pure radiation, and as $\mathcal{P}_0 \to \infty$, to the classical conduction equation. The linearized form of Eq. (17) for gray gas ($\zeta = 1$ and $\alpha_M = \alpha$) may be found in Goody.[5] Next, the unsteady slip condition is developed.

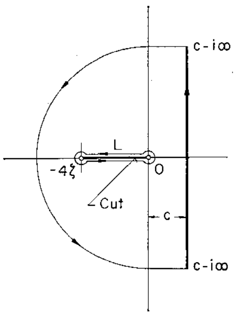

Fig. 2 Inversion contour.

For total values, Eqs. (6) and (7) may, respectively, be written as

$$\pi I^+(0+,t) = E_b(0+,t) + \tfrac{1}{2}q^R(0,t) - \tfrac{1}{4}\partial q^R(0,t)/\zeta\partial\tau \quad (19)$$

and

$$\pi I^+(0+,t) = E_b(-0,t) - (\rho/\epsilon)q^R(0,t) \quad (20)$$

From Eqs. (19) and (20) it follows that

$$4\lambda\zeta\sigma[T^4(-0,t) - T^4(0+,t)] = \zeta q^R(0,t) - \lambda\partial q^R(0,t)/\partial\tau \quad (21)$$

where $1/\lambda = 4(1/\epsilon - \frac{1}{2})$. Furthermore, Eq. (21) may be expressed conveniently in terms of the temperature alone. Elimination of q^R between Eqs. (15) and (21), and that of $\partial q^R/\partial\tau$ between this result and Eq. (16) with negligible conduction yields the unsteady temperature jump,

$$\left(\frac{\partial}{\partial\tau} - 3\,\frac{\lambda}{\zeta}\right)\frac{\partial\theta(0+,t)}{\partial t} + 3\lambda[\theta^4(-0,t) - \theta^4(0+,t)] + \zeta\,\frac{\partial\theta^4(0+,t)}{\partial\tau} = 0 \quad (22)$$

For the case of appreciable conduction, ignoring the jump and following the same steps, the boundary condition may be obtained in the form

$$\left(\frac{\partial}{\partial\tau} - 3\,\frac{\lambda}{\zeta}\right)\left(\frac{\partial}{\partial t} - \mathcal{P}_0\,\frac{\partial^2}{\partial\tau^2}\right)\theta(0,t) + \zeta\,\frac{\partial\theta^4(0,t)}{\partial\tau} = 0 \quad (23)$$

The linearized form of Eq. (23) for steady gray gas ($\partial/\partial t \equiv 0$, $\zeta = 1$, $\alpha_M = \alpha$) may be found in Goody.[5] It is worth noting that Eqs. (22) and (23), although developed in terms of a stagnant gas, equally apply to one-dimensional flows, provided the effect of compressibility and viscous dissipation are negligible. Furthermore, the modification of Eqs. (22) and (23) by these effects presents no difficulty (see Cogley[10] for an unsteady compressible problem including black walls).

Example

The foregoing development may be best illustrated in terms of a simple problem. Unsteady problems associated with semi-infinite stagnant conducting gray gas next to a black wall have already been explored by Nemchinov[11] and Lick.[12] However, the problems of nongray gas bounded by nongray walls so far remained untreated.

The particular problem in mind is a semi-infinite gas whose boundary temperature is suddenly changed to T_w from initial equilibrium temperature T_0 of the gas. Although the equation governing the case of negligible conduction is simpler than that of appreciable conduction, the unsteady temperature jump on the boundary of the former should make its solution comparable with, if not more complex than, that of the latter. Therefore, the case of negligible conduction deserves some attention, and is studied below.

The nonlinear terms associated with this problem preclude an analytical solution. However, since the interest lies in the physics rather than the accuracy of mathematics, these terms may be linearized by a Taylor expansion about the initial equilibrium temperature $T = T_0(\theta = 1)$. Then, in terms of $\psi = \theta - 1$, Eqs. (18) and (22),

$$(\partial^2/\partial\tau^2 - 3)\partial\psi/\partial t = -4\zeta\,\partial^2\psi/\partial\tau^2,\ \psi(\tau,0) = 0$$

$$\left(\frac{\partial}{\partial\tau} - 3\,\frac{\lambda}{\zeta}\right)\frac{\partial\psi(0+,t)}{\partial t} + 12\lambda[\psi_w - \psi(0+,t)] + 4\zeta\,\frac{\partial\psi(0+,t)}{\partial\tau} = 0$$

$\lim_{\tau\to\infty} \psi(\tau,t) \to 0$, where $\psi_w = \theta_w - 1$. Hereafter, without changing the notation already used for the Planck mean and the Rosseland mean of the absorption coefficient, these mean values will be interpreted as suggested by Cogley, Vincenti and Gilles,[13] and by Gilles, Cogley, and Vincenti.[14]

The foregoing formulation may be solved by Laplace transforms, also employed in Ref. 12. The transform of this formulation

$$d^2\bar{\psi}/d\tau^2 - [3p/(p + 4\zeta)]\bar{\psi} = 0$$

$$d\bar{\psi}(0+,p)/d\tau - (3\lambda/\zeta)\bar{\psi}(0+,p) + 12\lambda\psi_w/p(p + 4\zeta) = 0$$

$$\lim_{\tau\to\infty}\bar{\psi}(\tau,p) \to 0$$

has the solution

$$\frac{\bar{\psi}(\tau,p)}{\psi_w} = \frac{12\lambda\exp[-(3p)^{1/2}\tau/(p + 4\zeta)^{1/2}]}{p(p + 4\zeta)[3\lambda/\zeta + (3p)^{1/2}/(p + 4\zeta)^{1/2}]} \quad (24)$$

This equation has branch points at $z = 0$ and $z = -4\zeta$. It can be shown that the bracketed terms of the denominator cannot be made zero by any real or complex number, thereby eliminating the possibility of any pole. Thus, the usual inversion contour is reduced to the evaluation of an integral over the loop L (Fig. 2). However, to compare the relative complexity of the present solution with that of Ref. 12 which is only for small and large values of time, a similar approach is followed here.

Small-time solution

Since small $t \to$ large p, the binomial expansion yields

$$[3p/(p + 4\zeta)]^{1/2} \cong 3^{1/2}[1 - 2\zeta/(p + 4\zeta)]$$

and Eq. (24) may accordingly be rearranged as

$$\frac{\bar{\psi}(\tau,p)}{\psi_w} = \lambda^*\left(\frac{\gamma}{\delta}\right)e^{-\tau^*/2}\left[\frac{1}{p} + \frac{\gamma}{p(p + \delta)}\right]\frac{e^{\tau^*\zeta/(p+4\zeta)}}{(p + 4\zeta)} \quad (25)$$

where $\lambda^* = 2(3)^{1/2}\lambda$, $\tau^* = 2(3)^{1/2}\tau$, $\gamma = 4\zeta^2/(2\zeta + \lambda^*)$ and $\delta = 4\zeta(\zeta + \lambda^*)/(2\zeta + \lambda^*)$. Noting that

$$[e^{\tau^*\zeta/(p+4\zeta)}]/(p + 4\zeta) \to e^{-4\zeta t}I_0[2(\zeta\tau^* t)^{1/2}]$$

(see, for example, Campbell and Foster,[15] Pair 655.1), and employing the convolutive property of Laplace transforms, the inverse of Eq. (25) may readily be obtained as

$$\frac{\psi(\tau,t)}{\psi_w} = \left(\frac{\lambda^*}{\zeta + \lambda^*}\right)e^{-\tau^*/2}\left\{4\zeta\int_0^t e^{-4\zeta s}I_0[2(\zeta\tau^* s)^{1/2}]ds - \gamma e^{-\delta t}\int_0^t e^{-\gamma s}I_0[2(\zeta\tau^* s)^{1/2}]ds\right\} \quad (26)$$

where I_0 denotes the modified Bessel function of the first kind, of order zero. The Bessel integrals of this solution have been extensively studied and tabulated (see Arpaci,[16] pp. 353–365, and the references cited therein). Furthermore, the jump in boundary temperature is

$$\psi(0,t)/\psi_w = [\lambda^*/(\lambda^* + \zeta)](1 - e^{-\delta t})$$

Clearly, the complexity of Eq. (26) is comparable with that of small time solution obtained in Ref. 12 including the conduction. At first sight, this result appears surprising. Here, the algebraic difficulties introduced to the classical conduction problems by the "boundary temperature jump" related to the definition of heat transfer coefficient should be recalled. Since the expression for radiation jump is much more involved than for "conduction jump," the nature of Eq. (26) should be expected. Note that the nongrayness of gas and boundary has only parametric effects on this equation.

Large-time solution

The mathematical elaborations of Ref. 12 on the related problem are informative but hardly necessary. Clearly, an asymptotic expansion of $T(\tau,t)$ can be deduced from the behavior of $T(\tau,p)$ near its singularity with largest real part, $p = 0$ (see, for example, Ref. 16, pp. 419–420). Hence, noting, for large values of time (small values of p), that

$$[3p/(p + 4\zeta)]^{1/2} \cong (3p/4\zeta)^{1/2}(1 - p/8\zeta)$$

a first approximation to Eq. (24) may be written as

$$\bar{\psi}(\tau,p)/\psi_w = e^{-\tau^*/4(p/\zeta)^{1/2}}/p[1 + (\zeta p)^{1/2}/\lambda^*] \tag{27}$$

The inversion of this equation (see, for example, Pair 812 of Ref. 15) is

$$\frac{\psi(\tau,t)}{\psi_w} = \operatorname{erfc}\left[\frac{\tau^*}{8(\zeta t)^{1/2}}\right] - \exp\left(\frac{\lambda^*\tau^*}{4\zeta} + \frac{\lambda^{*2}t}{\zeta}\right) \times \operatorname{erfc}\left[\frac{\tau^*}{8(\zeta t)^{1/2}} + \lambda^*\left(\frac{t}{\zeta}\right)^{1/2}\right] \tag{28}$$

definitions of τ^* and λ^* remaining identical to those given in the small time solution. The jump in boundary temperature then follows

$$\psi(0,t)/\psi_w = 1 - \exp(\lambda^{*2}t/\zeta) \operatorname{erfc}[\lambda^*(t/\zeta)^{1/2}]$$

It is interesting to note that the approximate large time solution given by Eq. (28) is identical in form to the exact solution of the unsteady conduction in a semi-infinite solid whose ambient temperature assumes a step change. The complexity of Eq. (28) is again comparable with that of the related solutions of Ref. 12. The difference between the penetrations of radiation and conduction results in the "wavelike motion" discussed in Ref. 12. A similar behavior is not observed here because of the absence of conduction. Since the results of Ref. 12 hold for small $\mathcal{P}_0$ but not for $\mathcal{P}_0 = 0$, a direct comparison between this reference and the present study is not possible.

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Effects of a Nonrigidly Supported Ballast on the Dynamics of a Slender Body Descending Through the Atmosphere

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Introduction

Many atmosphere-entry bodies, particularly slender bodies, are ballasted for adequate aerodynamic static stability. The ballast typically is heavy, dense, and difficult to support in a perfectly rigid fashion under dynamic conditions. The purpose of this Note is to report some effects on the vehicle dynamics when the ballast is not rigidly supported

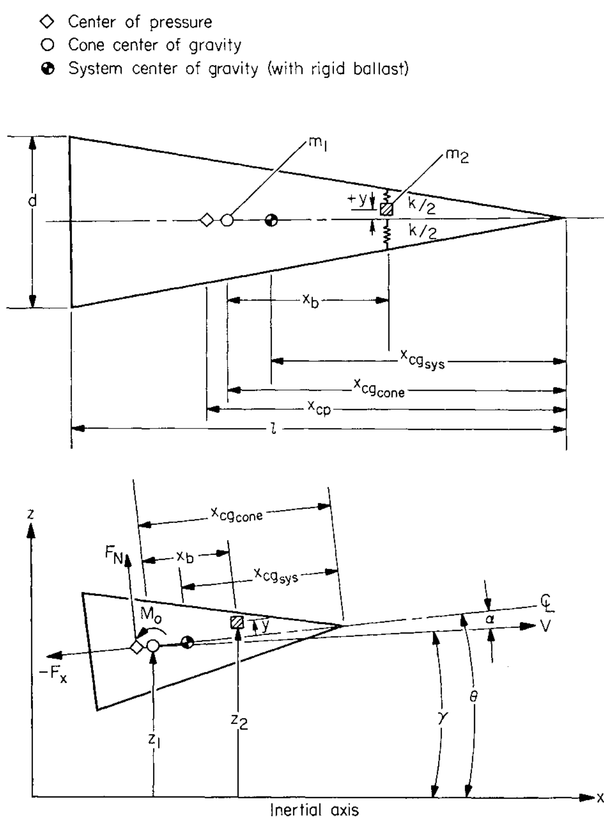

Fig. 1 Vehicle system.

Received June 17, 1970.

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